

# Microscopic description of light unstable nuclei with the stochastic variational method

K. Varga<sup>a</sup>, Y. Suzuki<sup>b</sup>, K. Arai<sup>c</sup>, and Y. Ogawa<sup>d</sup>

<sup>a</sup> Institute of Nuclear Research of the Hungarian Academy of Sciences, Debrecen, H-4001, Hungary

<sup>b</sup> Department of Physics, Niigata University  
Niigata 950-21, Japan

<sup>c</sup>Graduate School of Science and Technology,  
Niigata University, Niigata 950-21, Japan

<sup>d</sup>RIKEN, Hirosawa, Wako, Saitama 351-01 Japan

The structure of the light proton and neutron rich nuclei is studied in a microscopic multicluster model using the stochastic variational method. This approach enables us to describe the weakly bound nature of these nuclei in a consistent way. Applications for various nuclei  ${}^6\text{--}{}^9\text{Li}$ ,  ${}^7\text{Be}$ ,  ${}^8\text{B}$ ,  ${}^9\text{C}$ ,  ${}^9\text{--}{}^{10}\text{Be}$ ,  ${}^9\text{--}{}^{10}\text{B}$  presented. The paper discusses the relation of this model to other models as well as the possible extension for p and sd shell nuclei.

## 1. Introduction

The experimentally intensively studied unstable nuclei [1] have challenged the theoretical nuclear physicists. These nuclei exhibit a “nucleon halo”, a new form of nuclear matter, characterized by low and nonuniform density distribution. To describe this nucleon (proton or neutron) halo care must be taken on the proper description of the nuclear dynamics. The halo structure is mostly interpreted as two- or three-body phenomena, that is most of the theoretical models assume a stable core and one or two valence nucleons [2]. Although one can understand various properties of the halo nuclei in these phenomenological models, it is a natural aim to go beyond these approaches. While there are sophisticated techniques to tackle the two- and three-body problems, the solution of the nuclear many-body problem is still much too complicated. The small separation energies a characteristic property of the halo nuclei, require extra caution. One often has to reproduce as small as 100 keV energy differences (for example the proton separation energy of  ${}^8\text{B}$  or neutron separation energy of  ${}^{11}\text{Li}$ ). These tiny energies play a crucial role in determining the asymptotic form of the wave function and therefore the halo structure and the physical properties characterizing the halo nuclei (momentum distribution, interaction cross section) depends on them very much. A reliable description, therefore, should be able to give accurate energies and correct asymptotic behaviour. Another important

feature to be taken into account is the presence of correlations between nucleons. While the correlation of nucleons is a well known property in nuclear physics, it is somewhat surprising that there are reasonable experimental indications to assume the existence of correlated neutron pairs in such a low density nucleon matter as the halo of  ${}^6\text{He}$  or  ${}^{11}\text{Li}$ . The above mentioned specific conditions (correlation, extended nonuniform density distribution, importance of asymptotic part) make very difficult the application of models using single particle bases. Much of the success of the three-body description is due to their capability to treat these requirements by applying a wave function depending on *relative* variables. The antisymmetrization is, however, much more difficult in the case of relative coordinates and one often sacrifices the exact treatment of the Pauli principle in favor of simpler model, assuming structureless clusters or “core” (for example  ${}^9\text{Li}+n+n$ ). At this point, besides the validity of assuming a simplified core, another problem, the question of interaction between constituent particles appear.

Our approach tries to combine the advantages of different descriptions. A microscopic framework is used, that is we treat nucleonic degrees of freedom thereby avoiding the need of knowledge of core-nucleon or nucleus-nucleus interaction. We use relative coordinates to have a flexible coordinate-system to describe the dynamics and to eliminate any problem in connection with the center-of-mass motion. Correlated basis functions is used to answer the challenges of the weakly bound halo structure. The spatial part of the basis functions are chosen to be Gaussian form to facilitate the fully analytical calculation of the matrix elements. The Pauli principle is treated exactly.

## 2. Formalism

In our variational approach the basis functions are assumed to have the form [3]

$$\psi_{(LS)JMTM_T}(\mathbf{x}, A) = \mathcal{A}\{\phi_{int}e^{-\frac{1}{2}\mathbf{x}A\mathbf{x}}[\theta_L(\mathbf{x})\chi_S]_{LM}\eta_{TM_T}\}, \quad (1)$$

where  $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_{N-1})$  is a set of relative (Jacobi) coordinates, the operator  $\mathcal{A}$  is an antisymmetrizer, the function  $\theta_{LM_L}(\mathbf{x})$  represents the angular part of the wave function,  $\chi_{SM_S}$  is the spin function and  $\phi_{int}$  is the intrinsic function of the clusters. The angular part  $\theta_{LM_L}(\mathbf{x})$  is a vector coupled product of spherical harmonics of the relative coordinates. The spin of the clusters coupled to total spin S.

The assumption of clusters is based on physical motivations as well as practical considerations. The clustering in light nuclei has long been known and supported by various experimental facts as well as numerical simulations [4,5]. If one assumes Gaussian packet  $\varphi_s^\gamma(\mathbf{r}) = (2\gamma/\pi)^{3/4}\exp\{-\gamma(\mathbf{r} - \mathbf{s})^2\}$  single particle states to describe a nucleus and try to find the optimal positions of the centers of the Gaussian packet, that is try to find the configurations that minimize the total energy in a realistic potential, one observes the formation of various clusters of nucleons (alpha, triton,  ${}^3\text{He}$ , etc.). The experimental and theoretical research devoted to unstable nuclei is also in favor of this approach (e.g.  ${}^6\text{He}=\alpha+n+n$ ,  ${}^{11}\text{Li}={}^9\text{Li}+n+n$ , and so on). The wave function of the clusters are approximated by a single harmonic oscillator shell model configuration. If more accurate description of the internal structure of clusters is needed one can superpose shell model configurations [6]. Alternatively, one can divide the clusters into smaller entities, for example if the description of the triton with a single shell model configuration insufficient

one can describe it as  $p + n + n$ . The model assumes an equal harmonic oscillator size parameter  $\beta$  for the clusters.

The assumption of clusters with relatively simple description of their structure requires the usage of an effective nucleon-nucleon interaction, because a realistic interaction with a strong repulsive core would not render the clusters to be bound. The effective interaction used in this calculation (Minnesota interaction) contains a spin-isospin dependent central and a spin-orbit potential. The Coulomb interaction between protons is treated exactly.

The matrix elements appearing in the variational equation are calculated in a fully analytical way [3].

### 3. The stochastic variational method

The adequate choice of the nonlinear parameters (the elements of the  $(N-1) \times (N-1)$  matrix  $A$ ) is very important. As the trial function contains a large number of nonlinear parameters and one has to superpose many basis functions to get the energy minimum a direct optimization of the parameters is not suitable. The number of the different spin-isospin and partial wave channels further complicates the choice of the basis functions. Moreover, the basis functions are nonorthogonal and none of them are indispensable, any of them can be equally represented by some other choice. This property qualifies a random selection of the basis function by judging on their contribution to the energy. We set up the basis stepwise by randomly choosing  $A$  from a preset domain of the parameter space and increase the basis dimension by one if the energy gain by including the randomly selected basis element is larger than a preset value,  $\epsilon$ . This is repeated until the energy converges. This random selection of the nonlinear parameters gives very accurate energy and keeps the size of the basis feasible. We are able to solve  $N = 2-7$  body problems with this simple strategy. The results, as shown in Table 1., agree very well with those in the literature. We also mention that our calculations for different potentials and different physical systems (including Coulombic systems in atomic physics) proved to be as accurate as the most precise methods. The accuracy reached in these calculation justifies the application of this method for the field of unstable nuclei.

The spin-isospin and partial wave channels are also randomly selected eliminating any bias from the construction of the wave function. Several examples show that this random selection gives the same percentage of different components of the wave function as a more direct way (e.g. choosing appropriate basis states in all possible channels and diagonalizing in the resulted large space) and it does not lead to a false wave function.

### 4. The microscopic multicluster model

The method has been applied for various nuclei, such as  ${}^{6-8}\text{He}$  [10,11],  ${}^{6-9}\text{Li}$ ,  ${}^7\text{Be}$ ,  ${}^8\text{B}$ ,  ${}^9\text{C}$ ,  ${}^{9-10}\text{Be}$ ,  ${}^{9-10}\text{B}$  [12–14].

Recently, we have studied the mirror nuclei  ${}^9\text{C}$  and  ${}^9\text{Li}$  in a microscopic  $\alpha + {}^3\text{He} + p + p$  and  $\alpha + {}^3\text{H} + n + n$  four-cluster model [13]. The  ${}^7\text{Be}$ – ${}^7\text{Li}$  and  ${}^8\text{B}$ – ${}^8\text{Li}$  mirror two- and three-body subsystems are also investigated with the same effective interaction [15]. The calculated ground state energies, the radii, and the densities of the nucleons are in good agreement with the experimental data. The magnetic and quadrupole moments, except for the magnetic moments of  ${}^8\text{B}$  and  ${}^8\text{Li}$ , are also reproduced well. The quadrupole moments of

Table 1

Energies and rms radii of  $N$ -nucleon systems interacting via the Malfliet-Tjon potential.

| $N$                 | $(L, S)J^\pi$   | Method     | $E$ (MeV) | $\langle r^2 \rangle^{1/2}$ (fm) | $\mathcal{K}$ |
|---------------------|-----------------|------------|-----------|----------------------------------|---------------|
| 3                   | $(0, 1/2)1/2^+$ | Faddeev[7] | -8.25273  |                                  |               |
|                     |                 | SVM        | -8.2527   | 1.682                            | 80            |
| 4                   | $(0, 0)0^+$     | CRCG[8]    | -31.357   |                                  | 1000          |
|                     |                 | SVM        | -31.360   | 1.4087                           | 150           |
| 5                   | $(1, 1/2)3/2^-$ | VMC[9]     | -42.98    | 1.51                             |               |
|                     |                 | SVM        | -43.48    | 1.51                             | 500           |
| 6 ( $^6\text{He}$ ) | $(0, 0)0^+$     | VMC[9]     | -66.34    | 1.50                             |               |
|                     |                 | SVM        | -66.30    | 1.52                             | 800           |
| 7 ( $^7\text{Li}$ ) | $(1, 1/2)3/2^-$ | SVM        | -83.4     | 1.68                             | 1300          |

$^9\text{C}$  and  $^7\text{Be}$  are predicted to be  $-5.04 \text{ e fm}^2$  and  $-6.11 \text{ e fm}^2$ . The microscopic multicluster model predicts that the neutron skin thickness is about 0.4 fm in  $^8\text{Li}$  and  $^9\text{Li}$ , while the proton skin thickness is 0.5 fm in  $^8\text{B}$  and  $^9\text{C}$ . Comparing to the neutron skin thickness of 0.8 fm found in  $^6\text{He}$  and  $^8\text{He}$  [11], we conclude that these nuclei do not show pronounced halo structure.

The mirror nuclei  $^9\text{Be}$  and  $^9\text{B}$  have also been described in our model [14]. These nuclei are described in a three-cluster model comprising two  $\alpha$ -particles and a single nucleon. The three-body dynamics of the clusters is taken into account by including both of the possible arrangements,  $(\alpha\alpha)N$  and  $(N\alpha)\alpha$ , and by using all the relevant partial waves of the relative motion of the clusters. The ground state of  $^9\text{Be}$ , the only particle-bound state in the spectra of these nuclei, is calculated by using the stochastic variational method, while the other particle-unbound states are studied by the complex scaling method. The calculated spectra of  $^9\text{Be}$  and  $^9\text{B}$  are compared with experiment in Table 2. The theoretical level sequence in  $^9\text{Be}$  has a good correspondence with the observed spectrum. The second  $\frac{3}{2}^-$  resonance is obtained at 4.3 MeV excitation energy. The other calculations [16–18] also predict the  $\frac{3}{2}_2^-$  state. Although no such state is cited in Ref. [19], the calculated resonance may correspond to the state at 5.59 MeV mentioned in Ref. [20]. We get two broad overlapping resonances with  $\frac{7}{2}^-$  and  $\frac{9}{2}^+$  at about 6.5 MeV. This agrees with the conclusion of the recent experiments [20,21]. We could not find a resonance with  $\frac{1}{2}^-$  around 8 MeV excitation energy in accordance with Refs. [20,21], although such a state is parenthetically quoted in Ref. [19]. Instead of this a  $\frac{5}{2}^-$  resonance is obtained at 7.9 MeV, which agrees with the result of Refs. [16,17]. The spectrum of  $^9\text{B}$  is less known experimentally compared to that of  $^9\text{Be}$ . The calculated spectrum is similar to the one of  $^9\text{Be}$ . We can predict the energy and the width of several resonances in  $^9\text{B}$  with the same accuracy as the case of  $^9\text{Be}$ . For example, our calculation predicts a missing  $\frac{1}{2}^-$  state of a 1 MeV width at the excitation energy of 2.43 MeV, which is in agreement with the result of a recent  $^9\text{Be}(p, n)$  reaction [22] that located the  $\frac{1}{2}^-$  state at 2.83 MeV. Although no definitive spin assignment is made to the state at 2.788 MeV excitation energy [19], our calculation supports a  $\frac{5}{2}^+$  assignment rather than  $\frac{3}{2}^+$ . The first excited  $1/2^+$  state was not localized in the present study.

Table 2

Energies and widths of the unbound states in  ${}^9\text{Be}$  and  ${}^9\text{B}$ . The energy is from the three-body threshold. The spin and parity of the 3.065 MeV state of  ${}^9\text{B}$  is assumed to be  $\frac{5}{2}^+$ .

|                 | exp. <sup>a</sup> |                              |                                   | cal.            |                      |
|-----------------|-------------------|------------------------------|-----------------------------------|-----------------|----------------------|
|                 | $J^\pi$           | $E(\text{MeV}\pm\text{keV})$ | $\Gamma(\text{MeV}\pm\text{keV})$ | $E(\text{MeV})$ | $\Gamma(\text{MeV})$ |
| ${}^9\text{Be}$ | $3/2^-$           | -1.5735                      | —                                 | -1.431          | —                    |
|                 | $1/2^+$           | $0.111\pm 7$                 | $0.217\pm 10$                     |                 |                      |
|                 | $5/2^-$           | $0.8559\pm 1.3$              | $0.00077\pm 0.15$                 | 0.84            | 0.001                |
|                 | $1/2^-$           | $1.21\pm 120$                | $1.080\pm 110$                    | 1.20            | 0.46                 |
|                 | $5/2^+$           | $1.476\pm 9$                 | $0.282\pm 11$                     | 1.98            | 0.6                  |
|                 | $3/2^+$           | $3.131\pm 25$                | $0.743\pm 55$                     | 3.3             | 1.6                  |
|                 | $3/2_2^-$         | $4.02\pm 100^b$              | $1.33\pm 360$                     | 2.9             | 0.8                  |
|                 | $7/2^-$           | $4.81\pm 60^b$               | $1.21\pm 230$                     | 5.03            | 1.2                  |
|                 | $9/2^+$           | $5.19\pm 60^b$               | $1.33\pm 90$                      | 4.9             | 2.9                  |
|                 | $(1/2^-)$         | $6.37\pm 80$                 | $\sim 1.0$                        |                 |                      |
| ${}^9\text{B}$  | $5/2_2^-$         |                              |                                   | 6.5             | 2.1                  |
|                 | $3/2^-$           | 0.277                        | $0.00054\pm 0.21$                 | 0.30            | 0.004                |
|                 | $1/2^+$           | (1.9)                        | $\simeq 0.7$                      |                 |                      |
|                 | $5/2^-$           | $2.638\pm 5$                 | $0.081\pm 5$                      | 2.55            | 0.044                |
|                 | $1/2^-$           | $3.11^c$                     | 3.1                               | 2.73            | 1.0                  |
|                 | $5/2^+$           | $3.065\pm 30$                | $0.550\pm 40$                     | 3.5             | 1.2                  |
|                 | $3/2^+$           |                              |                                   | 4.6             | 2.7                  |
|                 | $3/2_2^-$         |                              |                                   | 4.2             | 1.4                  |
|                 | $7/2^-$           | $7.25\pm 60$                 | $2.0\pm 200$                      | 7.0             | 1.7                  |
|                 | $9/2^+$           |                              |                                   | 6.6             | 3.3                  |
|                 | $5/2_2^-$         |                              |                                   | 8.4             | 2.4                  |

a) Ref.[19].    b) Ref.[20].    c) Ref. [22].

The electromagnetic moments and the rms radii of proton, neutron, and nucleon, assuming pointlike nucleons, are included in Table 3. Bare operators are used in the calculation. The theory is found to reproduce the data very well. The fact that the total reaction cross section is reproduced well also supports that the calculated ground state density is reliable. The  $1/2^+ \rightarrow 3/2^-$   $E1$  transition and the  $5/2^- \rightarrow 3/2^-$   $E2/M1$  transitions are studied by treating the excited states as quasibound states. The calculated transition rates, shown in Table 3., are in good agreement with the experiments.

Table 3

Radii and electromagnetic properties of  ${}^9\text{Be}$ . The reduced matrix elements are given in Weisskopf units. The bare-nucleon charges and  $g$ -factors are used in the present calculation. The effective charges were used in the shell model calculation of Refs. [21] and [23] to calculate the quadrupole moment and the  $E2$  strength. See text for the  $B(E1)$  value of the present calculation.

| $J^\pi$ |                                                  | exp. <sup>a</sup>    | present      |
|---------|--------------------------------------------------|----------------------|--------------|
| $3/2^-$ | $E$ (MeV)                                        | -1.5735              | -1.431       |
|         | $r_m$ (fm)                                       |                      | 2.50         |
|         | $r_p$ (fm)                                       | $2.37 \pm 0.01$      | 2.39         |
|         | $r_n$ (fm)                                       |                      | 2.58         |
|         | $\mu$ ( $\mu_N$ )                                | $-1.1778 \pm 0.0009$ | -1.169       |
|         | $Q$ (e fm <sup>2</sup> )                         | $5.3 \pm 0.3$        | 5.13         |
|         | $\sigma_R$ (mb)                                  | $825 \pm 20^b$       | 850          |
| $5/2^-$ | $E$ (MeV)                                        | 0.8559               | 0.883        |
|         | $B(E2; \frac{5}{2}^- \rightarrow \frac{3}{2}^-)$ | $24.4 \pm 1.8$       | 22.0         |
|         | $B(M1; \frac{5}{2}^- \rightarrow \frac{3}{2}^-)$ | $0.30 \pm 0.03$      | 0.229        |
| $1/2^+$ | $E$ (MeV)                                        | 0.111                |              |
|         | $B(E1; \frac{1}{2}^+ \rightarrow \frac{3}{2}^-)$ | $0.22 \pm 0.09$      | 0.24<br>0.18 |

The fact that the present calculation reproduced all the data very well strongly supports that the three-cluster model is quite appropriate for describing the structure of  ${}^9\text{Be}$  and  ${}^9\text{B}$ , provided that the three-body dynamics is treated properly in the calculation. Because the ground state and the  $\frac{5}{2}^-$ , 2.43 MeV state are described well by the present model, the  $\beta$  decay of the  ${}^9\text{Li}$  ground state to these states is expected to further test the accuracy of their wave functions or an available wave function of  ${}^9\text{Li}$ . The experimental value of  $\log ft$  for the  $\beta$  decay to the  ${}^9\text{Be}$  ground state is about 5.31 [19,24], indicating that the  $\beta$ -decay matrix element is fairly suppressed despite the allowed transition. The weak  $\beta$  decay is ascribed to the fact that the spatial symmetry of the main component of  ${}^9\text{Be}$  is different from that of  ${}^9\text{Li}$  [26]. In fact the Gamow-Teller (GT) matrix element to any state of  ${}^9\text{Be}$ , if it is described by the  $\alpha + \alpha + n$  three-cluster model, always vanishes regardless of the wave function of  ${}^9\text{Li}$ . To explain the weak  $\beta$  decay we have to admix a small component which is not taken into account in the three-cluster model. This possibility has been investigated by including the distortion of the  $\alpha$ -particle into  $t + p$  and  $h + n$  configurations. The calculated  $\log ft$  value turns out to be 5.60, indicating that we are on the right track. By being able to accommodate such distortion into the model consistently we have exemplified a unique advantage of the microscopic multicluster model.

We have investigated the question of whether or not some high isospin excited states of stable nuclei have extended halo-like structure. For this we have focused on the  $0^+$  state of  ${}^6\text{Li}$  at 3.563 MeV excitation energy, which is the  $T_z=0$  member of the isospin triplet together with the ground states of  ${}^6\text{He}$  and  ${}^6\text{Be}$ . This indicates that the spatial structure of  ${}^6\text{Li}(0^+)$  is very similar to the  ${}^6\text{He}$  ground state which is known to have two-neutron

halo structure.

We have done microscopic calculations for  ${}^6\text{Li}(0^+)$  with  $\alpha+p+n$  three-body model by allowing the isospin mixing due to the Coulomb potential. A large number of channels was taken into consideration in the calculation. The isospin mixing was found to be moderate and does not destroy the isobaric analogue concept proposed in ref. [25] even near the three-particle threshold. The accuracy of the wave function was tested by calculating the electron scattering form factors. The calculated form factors are in fair agreement with experiment.

The neutron and proton density distributions of  ${}^6\text{Li}(0^+)$  are similar and more extended up to larger distance than the  ${}^6\text{He}$  density. The matter size of  ${}^6\text{Li}(0^+)$  is calculated to be 2.73 fm, which is by 0.2 fm larger than the  ${}^6\text{He}$  size. Our analysis strongly supports that the 3.563 MeV  $0^+$  state of  ${}^6\text{Li}$  has spatially extended halo-like structure formed by the neutron and proton outside the  $\alpha$ -particle. The inelastic proton [27] and pion [28] scatterings to this state show strong anomaly in the sense that any theoretical analysis leads to outstanding discrepancy from experiment. We think the consideration of the halo-like structure in this state is needed in such an analysis.

Further development of radioactive beam facilities will open up a spectroscopic study on high isospin excited states of stable nuclei as well as unstable nuclei near the drip-line and give us valuable information on the isospin impurity and the generalization of the neutron-halo concept.

## 5. Comparison to other models

As we have shown in a number of examples, the microscopic multicluster model provides us with a good and consistent description of light nuclei including unstable nuclei. The reason for success is that it can duly take into account the dynamical correlation between the nucleons as well as the asymptotic behavior characteristic to the weakly bound system. To treat the relative motion of the clusters flexibly enough, the trial function for the relative motion is chosen to be correlated Gaussians. The complexity of the wave functions is increased further if different cluster partitions are included in the model space.

The direct comparison of our wave function and model space to that of shell model or even to other versions of cluster models (which use different trial functions) is very complicated. Some way for comparison, however, would be inevitably necessary to have deeper understanding of applicability and limitations of different models. It would be useful to have a simple measure to compare various types of wave functions. We have recently proposed to use the occupation probability of the number of total oscillator quanta as a possible candidate as a tool of comparison[29]. The probability  $P_Q$  of a definite number of total HO quanta  $Q$  can be obtained by calculating the expectation value of the operator  $\mathcal{O}$

$$\mathcal{O} = \frac{1}{2\pi} \int_0^{2\pi} d\theta \exp\left(i\theta \left( \sum_{i=1}^A P_i \left[ H_{\text{HO}}(i) - \frac{3}{2} \right] - Q \right)\right), \quad (2)$$

where  $H_{\text{HO}}(i)$  is the 3-dimensional HO Hamiltonian divided by  $\hbar\omega = \frac{2\hbar^2}{m}\gamma$ .

$P_i$  projects out either proton or neutron. It is set the unit operator when one calculates the number of total quanta occupied by both protons and neutrons. The advantage of this formalism is that the evaluation of the matrix element of this operator is very simple in cluster models and it is trivial in shell model. Table 4. shows the examples for a pair of the mirror nuclei  ${}^7\text{Li}$ - ${}^7\text{Be}$ ,  ${}^8\text{Li}$ - ${}^8\text{B}$ , and  ${}^9\text{Li}$ - ${}^9\text{C}$ . The result for  ${}^6\text{He}$  and  ${}^9\text{Be}$  is also included in the table. The value of  $\gamma$  is set  $0.17 \text{ fm}^{-2}$  ( $\hbar\omega = 14.4 \text{ MeV}$ ). The probabilities are given as a function of  $Q_{\text{exc}} = Q - Q_{\text{min}}$ , where  $Q_{\text{min}}$  is the minimum number of HO quanta for the lowest Pauli-allowed configuration. The lowest  $0\hbar\omega$  component is around 50-60 % for most cases and the sum of 0, 2, and  $4\hbar\omega$  components accumulates to about 90 %. The admixtures of higher components than  $Q_{\text{exc}} = 4$  are significant in the ground states of  ${}^8\text{B}$  and  ${}^9\text{Be}$  and also in the ground state of  ${}^6\text{He}$ .

## 6. Extension to larger systems

The application of the SVM on correlated Gaussian basis for more than  $N=6-7$  body system is difficult as both the partial wave expansion and the calculation of the matrix elements of the Hamiltonian become too complicated and computer time consuming. Let us try to use some other type of basis functions to avoid these problems. The Gaussian packet  $\varphi_s^\gamma(\mathbf{r}) = (2\nu/\pi)^{3/4} \exp\{-\gamma(\mathbf{r} - \mathbf{s})^2\}$  functions are also often used in few-body and few-cluster calculations. These functions do not belong to a particular orbital angular momentum and if one uses them as single particle basis functions in a Slater determinant, the calculation of the matrix elements will not cause serious problems. In a variational calculation the basis functions take the form:

$$\Phi_s = \frac{1}{\sqrt{N!}} \det\{\varphi_{s_i}^{\nu_i}(\mathbf{r}_j)\} \quad (3)$$

The wave function of the system is approximated by linear combinations of these Slater determinants. States of good angular momenta can be obtained by letting an angular momentum projection operator act on the full wave function.

The adequate choice of the nonlinear parameters  $\{\nu_i, \mathbf{s}_i\}_{i=1}^N$  of the Slater determinants is very important. These nonlinear parameters can be selected by the stochastic variational method (SVM) or by an appropriate direct optimization. To keep the cost of the optimization low only a set of the nonlinear parameters in a given Slater determinant was optimized at a time, while those of the others were kept fixed.

The energy quite slowly converge on this uncorrelated basis and for few-body systems it fails to reach the same value as on the angular momentum projected Gaussian (see Table 5.). For heavier systems, where the angular momentum projected Gaussian basis is not feasible, the results on shifted Gaussian basis is quite close to those of other methods such as the Integrodifferential Equation Approach (IDEA) or the Hartree-Fock-Bogoljubov (HFB) method.

The Antisymmetrized Molecular Dynamics (AMD) [4] and Fermionic Molecular Dynamics (FMD) [5] methods use the shifted Gaussian basis functions and often approximate the wave function of a multinucleon system by only one Slater-determinant ( $K = 1$  in Table 5.). Table 5. shows that this approximation can be insufficient, and the linear combination of Slater determinants considerably lowers the ground state energy.



Table 4

The occupation probability of the number of harmonic-oscillator quanta for microscopic multicluster-model wave functions. The probabilities for nucleons, protons, and neutrons are given in % in the upper, middle, and lower rows, respectively, as a function of oscillator excitations. Asterisk indicates the probability of less than 1 % and dashed line represents vanishing probability. The average number of oscillator excitations is given in the column labeled  $\langle Q_{\text{exc}} \rangle$ . The details of the wave functions are referred to Ref. [12] for  ${}^6\text{He}$ , to Ref. [13] for  ${}^7\text{Li}$ ,  ${}^8\text{Li}$ ,  ${}^9\text{Li}$ , and  ${}^9\text{C}$ , and to Ref. [14] for  ${}^9\text{Be}$ .

| state<br>(model)       | rms radius<br>[fm] | $Q_{\text{exc}}$ |    |    |   |    |   |   |   |   |   |    |    | $\langle Q_{\text{exc}} \rangle$ |
|------------------------|--------------------|------------------|----|----|---|----|---|---|---|---|---|----|----|----------------------------------|
|                        |                    | 0                | 1  | 2  | 3 | 4  | 5 | 6 | 7 | 8 | 9 | 10 | 11 |                                  |
| ${}^6\text{He}(0^+)$   | $r_m = 2.51$       | 60               | —  | 14 | — | 12 | — | 5 | — | 3 | — | 2  | —  | 2.2                              |
| $(\alpha+n+n)$         | $r_p = 1.87$       | 74               | 10 | 11 | 2 | 1  | * | * | * | * | * | *  | *  | 0.5                              |
|                        | $r_n = 2.78$       | 67               | 3  | 8  | 5 | 7  | 2 | 2 | 1 | 1 | * | *  | *  | 1.7                              |
| ${}^7\text{Li}(3/2^-)$ | $r_m = 2.34$       | 63               | —  | 20 | — | 9  | — | 4 | — | 2 | — | *  | —  | 1.4                              |
| $(\alpha+t)$           | $r_p = 2.28$       | 77               | 2  | 16 | * | 4  | * | * | * | * | * | *  | *  | 0.6                              |
|                        | $r_n = 2.38$       | 73               | 1  | 17 | * | 5  | * | 1 | * | * | * | *  | *  | 0.8                              |
| ${}^7\text{Be}(3/2^-)$ | $r_m = 2.36$       | 62               | —  | 20 | — | 9  | — | 4 | — | 2 | — | 1  | —  | 1.6                              |
| $(\alpha+h)$           | $r_p = 2.41$       | 71               | 1  | 17 | * | 5  | * | 2 | * | * | * | *  | *  | 0.9                              |
|                        | $r_n = 2.31$       | 75               | 2  | 16 | * | 4  | * | 1 | * | * | * | *  | *  | 0.7                              |
| ${}^8\text{Li}(2^+)$   | $r_m = 2.45$       | 61               | —  | 18 | — | 11 | — | 4 | — | 2 | — | 1  | —  | 1.7                              |
| $(\alpha+t+n)$         | $r_p = 2.19$       | 79               | 6  | 11 | 1 | 2  | * | * | * | * | * | *  | *  | 0.4                              |
|                        | $r_n = 2.60$       | 67               | 3  | 14 | 2 | 7  | 1 | 2 | * | 1 | * | *  | *  | 1.3                              |
| ${}^8\text{B}(2^+)$    | $r_m = 2.63$       | 54               | —  | 17 | — | 12 | — | 6 | — | 3 | — | 1  | —  | 2.7                              |
| $(\alpha+h+p)$         | $r_p = 2.83$       | 60               | 3  | 14 | 3 | 8  | 2 | 3 | 1 | 2 | * | *  | *  | 2.1                              |
|                        | $r_n = 2.26$       | 74               | 9  | 12 | 2 | 2  | * | * | * | * | * | *  | *  | 0.6                              |
| ${}^9\text{Li}(3/2^-)$ | $r_m = 2.40$       | 66               | —  | 17 | — | 11 | — | 4 | — | 2 | — | *  | —  | 1.3                              |
| $(\alpha+t+n+n)$       | $r_p = 2.10$       | 82               | 6  | 9  | 1 | 1  | * | * | * | * | * | *  | *  | 0.4                              |
|                        | $r_n = 2.54$       | 71               | 3  | 12 | 2 | 6  | 1 | 2 | * | * | * | *  | *  | 1.0                              |
| ${}^9\text{C}(3/2^-)$  | $r_m = 2.52$       | 60               | —  | 17 | — | 12 | — | 5 | — | 3 | — | 1  | —  | 1.8                              |
| $(\alpha+h+p+p)$       | $r_p = 2.68$       | 65               | 4  | 12 | 3 | 7  | 1 | 2 | * | 1 | * | *  | *  | 1.4                              |
|                        | $r_n = 2.16$       | 79               | 8  | 9  | 2 | 1  | * | * | * | * | * | *  | *  | 0.4                              |
| ${}^9\text{Be}(3/2^-)$ | $r_m = 2.50$       | 54               | —  | 21 | — | 12 | — | 5 | — | 3 | — | 2  | —  | 2.1                              |
| $(\alpha+\alpha+n)$    | $r_p = 2.39$       | 71               | 3  | 17 | 1 | 5  | * | 1 | * | * | * | *  | *  | 0.8                              |
|                        | $r_n = 2.58$       | 65               | 2  | 18 | 1 | 8  | * | 3 | * | 1 | * | *  | *  | 1.3                              |

Table 5

The binding energies (in MeV) of different A-nucleon systems interacting via the Volkov potential ( $m = 0.6$ )

|                 | $K = 1$ | $K = 100$ | method | result |
|-----------------|---------|-----------|--------|--------|
| $^3\text{He}$   | -6.66   | -8.31     | SVM    | -8.46  |
| $^4\text{He}$   | -27.92  | -29.75    | SVM    | -30.42 |
| $^6\text{He}$   | -23.59  | -29.38    | SVM    | -31.82 |
| $^8\text{Be}$   | -52.60  | -57.09    |        |        |
| $^{16}\text{O}$ | -1100.1 | - - -     | IDEA   | -1101. |

## 7. Summary

In summary, we presented the new results of the microscopic multicluster model using the stochastic variational method. The stochastic variational method provides us with accurate energies and wave functions. We have determined physical quantities of interest (proton, neutron and matter distributions and radii, magnetic and quadrupole moments, electromagnetic transition rates, beta-decay probabilities, momentum distribution of fragments, interaction cross sections, spectroscopic amplitudes) for various light nuclei. We have discussed the relation of our approach to other models. Further applications for light nuclei, for example  $^{10}\text{Be}$ ,  $^{11}\text{Be}$ ,  $^{11}\text{Li}$  are under way.

This work was supported by Grant-in Aids for Scientific Research on Priority Areas (No. 05243102), for Scientific Research (C) (No. 0664038), and for International Scientific Research (Joint Research) (No. 08044065) of the Ministry of Education, Science and Culture (Japan) and by OTKA Grant No. T17298 (Hungary). Most of the calculations were done with the use of RIKEN's VPP500 computer.

## REFERENCES

1. I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, K. Sugimoto, O. Yamakawa, T. Kobayashi, and N. Takahashi, Phys. Rev. Lett. 55, 2676 (1985); I. Tanihata, T. Kobayashi, O. Yamakawa, S. Shimoura, K. Ekuni, K. Sugimoto, N. Takahashi, T. Shimoda, and H. Sato, Phys. Lett. B 206, 592 (1988).
2. M. V. Zhukov, B. V. Danilin, D. V. Fedorov, J. M. Bang, I. J. Thompson, and J. S. Vaagen, Phys. Rep. 231, 150 (1993).
3. K. Varga and Y. Suzuki, Phys. Rev C52 (1995) 2885, and Phys. Rev. A53 (1996) 1907;
4. A. Ono, H. Horiuchi, T. Maruyama and A. Ohnishi, Phys. Rev C47 (1993) C47;
5. H. Feldmeier, Nucl. Phys. A515 (1990) 147.
6. P. Descouvemont, Nucl. Phys. A581 (1995) 61
7. N. W. Schellingerhout and L. P. Kok, Nucl. Phys. A508 (1990) 299c.
8. M. Kamimura and H. Kameyama, Nucl. Phys. A508 (1990) 17c.
9. B. Wiringa, private communication.
10. K. Varga, Y. Suzuki, and R. G. Lovas, Nucl. Phys. A371 (1994) 447.

11. K. Varga, Y. Suzuki, and Y. Ohbayasi, Phys. Rev. C 50, 189 (1994).
12. K. Arai, Y. Suzuki, and K. Varga, Phys. Rev. C 51 (1995) 2488.
13. K. Varga, Y. Suzuki and I. Tanihata, Phys. Rev. C52 (1995) 3013;
14. K. Arai, Y. Suzuki, Y. Ogawa and K. Varga, Phys. Rev. C54 (1996) 132;
15. D. R. Thompson, M. Lemere, and Y. C. Tang, Nucl. Phys. A286, 53 (1977); I. Reichstein and Y. C. Tang, *ibid.* A158, 529 (1970).
16. S. Okabe, Y. Abe, and H. Tanaka, Prog. Theor. Phys. 57, 866 (1977); S. Okabe and Y. Abe, *ibid.* 59, 315 (1978); *ibid.* 61, 1049 (1979).
17. H. Furutani, H. Kanada, T. Kaneko, S. Nagata, H. Nishioka, S. Okabe, S. Saito, T. Sakuda, and M. Seya, Prog. Theor. Phys. Suppl. 68, 193 (1980).
18. P. Descouvemont, Phys. Rev. C 39, 1557 (1989).
19. F. Ajzenberg-Selove, Nucl. Phys. A490, 1 (1988).
20. S. Dixit *et al.*, Phys. Rev. C 43, 1758 (1991).
21. J. P. Glickman *et al.*, Phys. Rev. C 43, 1740 (1991).
22. B. Pugh quoted in: M. A. Tiede *et al.*, Phys. Rev. C 52, 1315 (1995).
23. A. G. M. van Hees and P. W. M. Glaudemans, Z. Phys. A— Atoms and Nuclei 315, 223 (1984); A. G. M. van Hees, A. A. Wolters, and P. W. M. Glaudemans, Nucl. Phys. A476, 61 (1988).
24. G. Nyman *et al.*, Nucl. Phys. A510, 189 (1990).
25. Y. Suzuki and K. Yabana, Phys. Lett. B272, 173(1991).
26. S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
27. F. Petrovich, S. K. Yoon, M. J. Threapleton, R. J. Philpott, and J. A. Carr, Nucl. Phys. A563, 387(1993).
28. R. R. Kiziah, M.D. Brown, C. J. Harvey, D. S. Oakley, W. B. Cottingham, R. W. Garnett, Steven J. Greene, and D. B. Holtkamp, Phys. Rev. C30, 1643(1984).
29. Y. Suzuki, K. Arai, Y. Ogawa, and K. Varga, submitted to Phys. Rev. C.